UNIVERSIDADE BEIRA INTERIOR

Departamento de Matemática

Formulário de Cálculo

Função Logarítmica

$\alpha \in \mathbb{R}, \quad a, b \in]0, +\infty[\setminus \{1\} \quad$ e $\quad x, y \in]0, +\infty[$

$\log_a (xy) = \log_a x + \log_a y$

$\log_a \dfrac{1}{x} = -\log_a x$

$\log_a \dfrac{x}{y} = \log_a x - \log_a y$

$\log_a (x^\alpha) = \alpha \log_a x$

$\log_a x = \log_b x \log_a b$

$\log_a 1 = 0$

Funções Trigonométricas

$\operatorname{sen}^2 x + \cos^2 x = 1$

$1 + \operatorname{tg}^2 x = \dfrac{1}{\cos^2 x}$

$1 + \operatorname{cotg}^2 x = \dfrac{1}{\operatorname{sen}^2 x}$

$\operatorname{sen}(2x) = 2 \operatorname{sen} x \cos x$

$\cos(2x) = \cos^2 x - \operatorname{sen}^2 x$

$\cos(2x) = 2\cos^2 x - 1$

$\cos(2x) = 1 - 2\operatorname{sen}^2 x$

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sen	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tg	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	n.d.
cotg	n.d.	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0

$\operatorname{sen}(x+y) = \operatorname{sen} x \cos y + \operatorname{sen} y \cos x$

$\operatorname{sen}(x-y) = \operatorname{sen} x \cos y - \operatorname{sen} y \cos x$

$\cos(x+y) = \cos x \cos y - \operatorname{sen} x \operatorname{sen} y$

$\cos(x-y) = \cos x \cos y + \operatorname{sen} x \operatorname{sen} y$

$\operatorname{sen} x + \operatorname{sen} y = 2 \operatorname{sen} \dfrac{x+y}{2} \cos \dfrac{x-y}{2}$

$\operatorname{sen} x - \operatorname{sen} y = 2 \operatorname{sen} \dfrac{x-y}{2} \cos \dfrac{x+y}{2}$

$\cos x - \cos y = -2 \operatorname{sen} \dfrac{x+y}{2} \operatorname{sen} \dfrac{x-y}{2}$

$\cos x + \cos y = 2 \cos \dfrac{x+y}{2} \cos \dfrac{x-y}{2}$

Funções Trigonométricas Inversas

	Domínio	Contradomínio	Definição
arc sen	$[-1, 1]$	$[-\pi/2, \pi/2]$	$\operatorname{arc\,sen} x = y \Leftrightarrow x = \operatorname{sen} y \ \wedge \ y \in [-\pi/2, \pi/2]$
arc cos	$[-1, 1]$	$[0, \pi]$	$\operatorname{arc\,cos} x = y \Leftrightarrow x = \cos y \ \wedge \ y \in [0, \pi]$
arc tg	$\mathbb{R}$	$]-\pi/2, \pi/2[$	$\operatorname{arc\,tg} x = y \Leftrightarrow x = \operatorname{tg} y \ \wedge \ y \in]-\pi/2, \pi/2[$
arc cotg	$\mathbb{R}$	$]0, \pi[$	$\operatorname{arc\,cotg} x = y \Leftrightarrow x = \operatorname{cotg} y \ \wedge \ y \in]0, \pi[$

Limites

$\lim\limits_{y \to 0} \dfrac{e^y - 1}{y} = 1$

$\lim\limits_{y \to 0} \dfrac{\ln(1+y)}{y} = 1$

$\lim\limits_{y \to 0} \dfrac{\operatorname{sen} y}{y} = 1$

$\lim\limits_{y \to 0} \dfrac{\operatorname{tg} y}{y} = 1$

$\lim\limits_{y \to 0} \dfrac{\operatorname{arc\,sen} y}{y} = 1$

$\lim\limits_{y \to 0} \dfrac{\operatorname{arc\,tg} y}{y} = 1$

$\lim\limits_{y \to +\infty} \operatorname{arc\,tg} y = \dfrac{\pi}{2}$

$\lim\limits_{y \to -\infty} \operatorname{arc\,tg} y = -\dfrac{\pi}{2}$

$\lim\limits_{y \to +\infty} \left(1 + \dfrac{a}{y}\right)^y = e^a$

Tabela de Derivadas

$\alpha \in \mathbb{R}$ e $a \in]0, +\infty[\setminus \{1\}$

$$[\alpha\, u(x)]' = \alpha\, u'(x)$$

$$[u(x) + v(x)]' = u'(x) + v'(x)$$

$$[u(x)\, v(x)]' = u'(x)\, v(x) + u(x)\, v'(x)$$

$$\left[\frac{u(x)}{v(x)}\right]' = \frac{u'(x)\, v(x) - u(x)\, v'(x)}{[v(x)]^2}$$

$$[(u(x))^\alpha]' = \alpha\, u'(x)\ [u(x)]^{\alpha-1}$$

$$\left[\sqrt{u(x)}\right]' = \frac{u'(x)}{2\sqrt{u(x)}}$$

$$\left[e^{u(x)}\right]' = u'(x)\, e^{u(x)}$$

$$[\ln(u(x))] = \frac{u'(x)}{u(x)}$$

$$\left[a^{u(x)}\right]' = u'(x) a^{u(x)}\ \ln a$$

$$[\log_a(u(x))]' = \frac{u'(x)}{u(x) \ln a}$$

$$[\operatorname{sen}(u(x))]' = u'(x)\ \cos(u(x))$$

$$[\cos(u(x))]' = -u'(x)\ \operatorname{sen}(u(x))$$

$$[\operatorname{tg}(u(x))]' = \frac{u'(x)}{\cos^2(u(x))} \rightarrow \sec^2$$

$$[\operatorname{cotg}(u(x))]' = -\frac{u'(x)}{\operatorname{sen}^2(u(x))}$$

$$[\operatorname{arc\,sen}(u(x))]' = \frac{u'(x)}{\sqrt{1-(u(x))^2}}$$

$$[\operatorname{arc\,cos}(u(x))]' = -\frac{u'(x)}{\sqrt{1-(u(x))^2}}$$

$$[\operatorname{arc\,tg}(u(x))]' = \frac{u'(x)}{1+(u(x))^2}$$

$$[\operatorname{arc\,cotg}(u(x))]' = -\frac{u'(x)}{1+(u(x))^2}$$

$$[\operatorname{senh}(u(x))]' = u'(x) \cosh(u(x))$$

$$[\cosh(u(x))]' = u'(x) \operatorname{senh}(u(x))$$

Tabela de Primitivas

$\alpha \in \mathbb{R} \setminus \{-1\}$ e $a > 0$

$$\int u'(x)\, [u(x)]^\alpha\ dx = \frac{[u(x)]^{\alpha+1}}{\alpha+1} + C,$$

$$\int \frac{u'(x)}{u(x)}\, dx = \ln|u(x)| + C \qquad \alpha = -1$$

$$\int u'(x)\, e^{u(x)}\ dx = e^{u(x)} + C$$

$$\int u'(x) a^{u(x)}\, dx = \frac{a^{u(x)}}{\ln a} + C \qquad \text{com } a \neq 1$$

$$\int u'(x) \operatorname{sen}[u(x)]\ dx = -\cos[u(x)] + C$$

$$\int u'(x) \cos[u(x)]\ dx = \operatorname{sen}[u(x)] + C$$

$$\int u'(x) \operatorname{senh}[u(x)]\ dx = \cosh[u(x)] + C$$

$$\int u'(x) \cosh[u(x)]\ dx = \operatorname{senh}[u(x)] + C$$

$$\int \frac{u'(x)}{\cos^2[u(x)]}\, dx = \operatorname{tg}[u(x)] + C$$

$$\int \frac{u'(x)}{\operatorname{sen}^2[u(x)]}\, dx = -\operatorname{cotg}[u(x)] + C$$

$$\int \frac{u'(x)}{\sqrt{a^2 - [u(x)]^2}}\, dx = \operatorname{arc\,sen}\frac{u(x)}{a} + C$$

$$\int \frac{u'(x)}{a^2 + [u(x)]^2}\, dx = \frac{1}{a} \operatorname{arc\,tg}\frac{u(x)}{a} + C$$

Primitivação e Integração por Partes

$$\int f'(x)\, g(x)\, dx = f(x)\, g(x) - \int f(x)\, g'(x)\, dx$$

$$\int_a^b f'(x)\, g(x)\, dx = \Big[f(x)\, g(x) \Big]_a^b - \int_a^b f(x)\, g'(x)\, dx$$

- Função Par: $f(-x)=f(x)$
- função Impar: $f(-x)=-f(x)$
- função composta: $(f\circ g)(x)=f(g(x))$

$D_{f\circ g}=\{x\in D_f : f(x)\in D_g\}$

$D_{f\circ g}=\{x\in \mathbb{R} : x\in D_f \wedge f(x)\in D_g\}$ → se a integral for impar $=0$

$|x|=\begin{cases} x & se\ x\geq 0 \\ -x & se\ x<0\end{cases}$

$|x|<a \Leftrightarrow -a<x<a$

$|x|>a \Leftrightarrow x>a \vee x<-a$

$f(x)=\begin{cases} x\,sen\frac{1}{x} & se\ x\neq 0 \\ 0 & se\ x=0\end{cases}$

$\lim_{x\to 0^+}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0^+}\frac{x\,sen(1/x)}{x}=\lim_{x\to 0^+} sen\frac{1}{x}$

$\lim_{x\to 0^-}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0^-} sen\frac{1}{x}$

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Cálculo I
Ano Letivo 2022/2023

FACULDADE CIÊNCIAS

Departamento de Matemática

Nome:

Nº:

- $(f(g))'=f'(g)\times g'$
- $\left(\frac{f}{g}\right)'=\frac{f'g-fg'}{g^2}$
- $(f\times g)'=f'g+fg'$
- $(f^n)'=n\times f^{n-1}\times f'$

$\boxed{\int_a^b f'(x)=f(b)-f(a)}$

$\int \frac{u'}{u}=\ln(u)+C$; $\int u'u^n=\frac{u^{n+1}}{n+1}+C$

$x\to$ ∪ ; $-x\to$ ∩

Derivadas

$(\cot g\, u)'=\frac{-u'}{sen^2 u}$

- $(\sqrt[n]{u})'=\frac{u'}{n\sqrt[n]{u^{n-1}}}\to(\sqrt{x})'=\frac{1}{2\sqrt{x}}$
- $(\ln x)'=\frac{1}{x}$ → Domínio: $\log_a(d(x))$, $d(x)>0$
- $(\ln u)'=\frac{u'}{u}$ → $\ln 1=0$; $\ln e=1$; $\log\frac{1}{x}=\log x^{-1}=-\log x$
- $(sen\, u)'=u'\cos u$
- $(\cos u)'=-u'\,sen\, u$
- $(\tan u)'=u'\sec^2 u$ → $(\tan u)'=\frac{u'}{\cos^2(u)}$
- $(\cotan u)'=-u'\,cosec^2 u \to -\frac{u'}{sen^2 u}$
- $(\sec u)'=\tan u\times\sec u$
- $(arcsen\, u)'=\frac{u'}{\sqrt{1-u^2}}$
- $(arccos\, u)'=\frac{-u'}{\sqrt{1-u^2}}$
- $(\sinh u)'=u'\cosh u$
- $(\cosh u)'=u'\sinh u$
- $(arctg\, u)'=\frac{u'}{1+u^2}$
- $(arcotan\, u)'=\frac{-u'}{1+u^2}$
- $(e^u)'=u'e^u$; $e^0=1$; $e^{\ln(\frac{1}{t})}=\frac{1}{t}$

$(abc)'=a'bc+ab'c+abc'$

Primitivas $+c,\ c\in\mathbb{R}$

- $\int 1\,dx=x+c$
- $\int x^\alpha dx=\frac{x^{\alpha+1}}{\alpha+1}$
- $\int e^x dx=1e^x$
- $\int \cos x\,dx=\sin x+c \to \int u'\cos u=sen\,u+c$
- $\int \frac{1}{x}dx=\ln|x|$
- $\int \sin x\,dx=-\cos x \to \int u'\cos u=sen\,u$
- $\int_a^a f(x)\,dx=0$
- $\int_b^a f(x)\,dx=-\int_a^b f(x)$ | $\int \frac{1}{2}f(x)=\frac{x}{2}$
- $\int x\,dx=\frac{x^2}{2}$

$\int_a^b f(x)dx=F(b)-F(a)$

Regra de Barrow

$\boxed{\int\frac{u'}{u}=\ln u+C}$

- $\int\frac{u'}{a^2+u^2}=\frac{1}{a}arctg\frac{u}{a}$
- $\int\frac{u'}{\sqrt{a^2-u^2}}=arcsen\frac{u}{a}$
- $\int u'\sec^2 u=tg\,u$
- $\int u'\,cosec^2 u=-\cot g\,u$
- $\int u'\sec u=\ln(\sec u+tg\,u)$
- $\int u'\,cosec\,u=\ln(cosec\,u+\cot g\,u)$
- $\int u'\sec u\,tg\,u=\sec u$
- $\int u'\,cosec\,u=-cosec\,u$

$arctg(-\infty)=-\frac{\pi}{2}$; $arctg(0)=0$; $arctg(1)=\frac{\pi}{4}$

Domínio tang: $\tan(d(x))$, $d(x)\neq\frac{\pi}{2}+k\pi$, $k\in\mathbb{Z}$

Casos notáveis

- $(a-b)^2=a^2-2ab+b^2$
- $(a+b)^2=a^2+2ab+b^2$
- $(a+b)(a-b)=a^2-b^2$
- $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
- $\sin\theta=\frac{op}{h}$; $\cos\theta=\frac{adj}{h}$; $\tan\theta=\frac{op}{adj}$

Formulas trigonométricas

- $\sin^2 x+\cos^2 x=1$
- $1+tg^2 x=\sec^2 x$
- $\cosh^2 x-\sinh^2 x=1$
- $tg\,x=\frac{\sin x}{\cos x}$
- $1+\cot g^2 x=cosec^2 x$
- $\cos(2x)=\cos^2 x-\sin^2 x$
- $\cos^2 x=\frac{1+\cos(2x)}{2}$
- $\tan^2 x\neq 1=\frac{1}{\cos^2 x}$

arcsen: $[-1,1]\to\mathbb{R}$, $y=\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

arccos: $[-1,1]\to\mathbb{R}$, $y=[0,\pi]$

- $sen(2x)=2\,sen\,x\cos x$
- $sen^2 x=\frac{1-\cos(2x)}{2}$
- $\sec x=\frac{1}{\cos x}$
- $cosec\,x=\frac{1}{\sin x}$

- $\sin(arccos\,x)=\sqrt{1-x^2}$; $\cos(arcsin\,x)=$
- $\cot g(arcsen\,x)=\frac{\sqrt{1-x^2}}{x}$; $\tan(arccos\,x)=$
- $\sin(arctg\,x)=\frac{x}{\sqrt{1+x^2}}$; $\tan(arcsin\,x)=$
- $sen\left(arccos\left(\frac{1}{x}\right)\right)+c=\frac{\sqrt{x^2-1}}{x}+c$
- $\cot g\,x=\frac{\cos x}{sen\,x}$
- $\cot g\,x=\frac{1}{\tan x}$
- $\sec\left(arctg\left(\frac{x}{a}\right)\right)=\frac{\sqrt{a^2+x^2}}{a}$

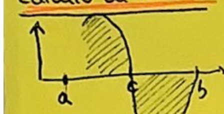
θ		$sen\,\theta$	$\cos\theta$	$\tan\theta$
30°	$\pi/6$	$1/2$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\pi/4$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\pi/3$	$\sqrt{3}/2$	$\frac{1}{2}$	$\sqrt{3}$

Integral à Riemann

- Area minorante: $m_i(x_i-x_{i-1})\to s(f,P)=\sum_{i=1}^n m_i(x_i-x_{i-1})$
- Area majorada: $M_i(x_i-x_{i-1})\to S(f,P)=\sum_{i=1}^n M_i(x_i-x_{i-1})$
- Função integrável à Riemann: $s(f,P)\leq A\leq S(f,P)$ (todas as funções contínuas são integráveis à Riemann)

Calculo de áreas

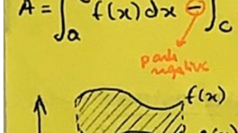
→ Interseção: $g(x)=f(x)$



$A=\int_a^c f(x)dx-\int_c^b f(x)dx$ (parte negativa)

Ex: $A=\int_0^1 \underbrace{(2-x)}_{\text{reta de cima}}-\underbrace{x}_{\text{reta de baixo}}\,dx$

⚠ Determinar os pontos de interseção para saber $\int_a^b$



$A=\int_a^b f(x)-g(x)\,dx$

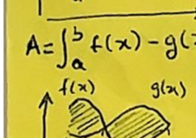
Ex: $y=2-x$; $x=0$, $y=x$

$\begin{cases}y=x\\ y=2-x\end{cases}\Leftrightarrow\begin{cases}x=1\\ y=1\end{cases}$

⚠ $x^2=x+2 \Leftrightarrow x^2-x-2=0$

C.A. $x^2-x-2=0$ $\Leftrightarrow x=2 \vee x=-1$

$y=(-1)=$; $y=2=$; $\begin{cases}y=x+2\\ y=x^2\end{cases}$



$A=\int_a^c f(x)-g(x)dx+\int_c^b g(x)-f(x)dx$

Ex: ⚠ $\int\frac{1}{1+t}\times\frac{1}{t}dt=\int\left(\frac{1}{t}-\frac{1}{t+1}\right)dt$; $\ln(t)$; $\ln(t+1)$

$t=e^{-x}$ → $x=\ln\left(\frac{1}{t}\right)$; $dx=\left(\ln\left(\frac{1}{t}\right)\right)'dt=-\frac{1}{t}dt$

$\int\frac{1}{1+\frac{1}{t}}\left(-\frac{1}{t}\right)dt=\int\frac{-1}{t+1}dx$

$=-\ln(e^x+1)+c=\ln(e^x+1)^{-1}+c=\ln\left(\frac{e^x}{\ldots}\right)$

$\int\left(\frac{1}{1+t}\times\frac{1}{t}\right)dx=\ln(1+t)+c$

Primitivação por partes

$\int \underbrace{f'(x)}_{u'}\underbrace{g(x)}_{v}dx=\underbrace{f(x)}_{u}\underbrace{g(x)}_{v}-\int\underbrace{f(x)}_{u}\underbrace{g'(x)}_{v'}dx$

$v'=e^{3x-1}$; $\int e^{3x-1}=v$

⚠ $(f(x)g(x))'=f'(x)g(x)+f(x)g(x)'$

Ex: $\int\ln x\,dx=\int 1\times\ln x\,dx=x\ln x-\int x(\ln x)'dx=x\ln x-\int x\times\frac{1}{x}$
$=x\ln x-x+c$, $c\in\mathbb{R}$

- Quadrados: $\int sen^2 x\,dx=\int \underbrace{sen\,x}_{u'}\times\underbrace{sen\,x}_{v}$
- $\int arctg\,x\,dx=\int 1\times arctg\,x\,dx=x\,arctg\,x-\int\frac{x}{1+x^2}dx$
$=x\,arctg\,x-\frac{1}{2}\int\frac{2x}{1+x^2}dx=x\,arctg\,x-\frac{1}{2}\ln(1-x^2)+c$
- $\int\cos(2x)dx=\frac{1}{2}\int 2\cos(2x)dx=\frac{1}{2}sen(2x)=\frac{1}{2}\times 2\,sen\,x\cos x$; $sen\,x\cos x$

$\boxed{\int_a^b f'(x)g(x)dx=\left[f(x)g(x)\right]_a^b-\int_a^b f(x)g'(x)dx}$

→ soma | subst. separa-se ; multiplicação - I. por partes

Primitivação por substituição

$f(x)=\int f(x)dx=\int f(\varphi(t))\times\varphi'(t)dt$ | $t=\varphi^{-1}(x)$

$x=2tg\,t$ → $dx=2\sec^2 t$

m.d.v → $x=\varphi(t)$; $dx=\varphi'(t)dt$

$\int_{g(c)}^{g(d)} f(x)dx=\int_c^d f(g(t))g'(t)dt$ → Quando $x=a$, $t=\ldots$; Quando $x=b$, $t=\ldots$

Propriedades dos Integrais

- $\int_a^b k\,dx = k(b-a)$
- $\int_a^b f(x)+g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$
- $\int_a^a f(x)\,dx = 0$
- $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$
- $\int_a^b k f(x)\,dx = k\int_a^b f(x)\,dx$
- $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ se $c \in [a,b]$

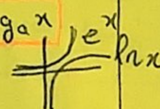
Não existem:
→ $\lim_{x\to\pm\infty} \operatorname{sen} x$
→ $\lim_{x\to\pm\infty} \cos x$

Limites Notáveis

- $\lim_{x\to 0} \frac{\operatorname{sen} x}{x} = 1$
- $\lim_{x\to+\infty} \left(1+\frac{1}{x}\right)^x = e$
- $\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$
- $\lim_{x\to 0} \frac{\ln(x+1)}{x} = 1$
- $\lim_{x\to 0} \frac{\operatorname{tg} x}{x} = 1$
- $\lim_{x\to+\infty} \frac{e^x-1}{x} = 1$
- $\lim_{x\to+\infty} \frac{\ln x}{x} = 0$
- $\lim_{x\to 0} \frac{\operatorname{arcsen} x}{x} = 1$
- $\lim_{x\to 0} \frac{\operatorname{arctg} x}{x} = 1$

Propriedades

- $a^x = y \Rightarrow x = \log_a y$
- $\ln x = y \Rightarrow x = e^y$
- $\log_a(x^y) = y\log_a x$
- $a^x = e^{x\ln a}$
- $a^{\log_a x} = x$
- $\log_a\left(\frac{1}{x}\right) = -\log_a x$
- $\log_a x = \frac{\log_b x}{\log_b a}$
- $a^x = b^{x\log_b a}$



Assintotas verticais

- Calcular D_f:
  - $D_f = \mathbb{R}$ – contínua, ñ tem
  - $D_f = \mathbb{R}\setminus\{a\}$ → a é ponto aderente
- $\lim_{x\to a^\pm} f(x)$ → se der $\pm\infty$ em algum, então ñ há $x=a$ A.V

Assintotas horizontais

$m=0$ $\quad y=$ __

$\lim_{x\to\pm\infty} f(x)$

Assintotas ñ verticais

$\lim_{x\to+\infty}[f(x)-(mx+b)]=0$

→ $D_f =]a,b[$ → ñ tem

→ Calcular D_f:

- $D_f = \mathbb{R}\setminus\{a\}$ ou $\mathbb{R}$ ou $]-\infty,a[$ ou $]a,+\infty[$

$$m = \lim_{x\to\pm\infty} \frac{f(x)}{x}$$

$$b = \lim_{x\to\pm\infty} [f(x)-mx]$$

→ substituir: $y = mx+b$

derivada num ponto

$f'(0) = \lim_{x\to 0} \frac{f(x)-f(0)}{x-0}$

- $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ $\quad 0^+$, 0^-

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

→ declive da reta:
$(a, f(a))$
$(a+h, f(a+h))$

Se $f'_e(x) \neq f'_d(x)$, f ñ é diferenciável

$y = f(a) + f'(a)(x-a)$

Teorema de Bolzano

Uma vez que a função é contínua no intervalo $[a,b]$ e $f(a)\neq f(b)$, podemos concluir que, para qualquer valor k, $\exists c \in [a,b] : f(c) = k$

COROLÁRIO: $f:[a,b]\to\mathbb{R}$

$f(a)\times f(b) < 0$
logo $\exists c\in[a,b] : f(c) = 0$ Tem zeros?

Teorema de Rolle

$f:[a,b]\to\mathbb{R}$
- Uma função contínua em $[a,b]$
- É diferenciável em $]a,b[$
- Se $f(a)=f(b)$, então existe $c\in\,]a,b[$ tal que $f'(c)=0$

Ex: $e^x = 3x$
- $f(x) = e^x - 3x$ → contínua
  $f(0)>0$; $f(1)<0$
  O T. de Rolle permite-nos concluir que f tem pelo menos 1 zero em $]0,1[$
- $f'(x) = (e^x-3x)' = e^x-3$ $\quad$ Nº de zeros
  $f'(x) = 0$
  $e^x = 3$ → tem 2 zeros, 2 soluções
  $\Leftrightarrow x = \ln 3$

Teorema de Taylor

aproximação linear

$$\underbrace{f(x)}_{P_{n,a}} = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots \frac{f^{(n)}(a)}{n!}(x-a)^n$$

aproximação quadrática

$$R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$$

→ Se $a=0$ → chama-se fórmula de Mac-Laurin

Ex: $f^{(n)}(x) = e^x$
$f^{(n)}(0) = e^0 = 1$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2} + \dots \frac{f^{(n-1)}(0)}{(n-1)!}x^{n-1} + \frac{f^{(n)}(0)}{n!}x^n = 1 + x + \frac{x^2}{2!} + \dots \frac{x^{n-1}}{(n-1)!} + \frac{x^n}{n!}$$

→ aproximação linear:
$e^x \approx 1+x$

→ aproximação quadrática:
$e^x \approx 1 + x + \frac{x^2}{2}$

Ex: Provar que se intersetam apenas 1 vez em $[3,6]$:

As funções intersetam-se no intervalo $[3,6]$ se $\exists c$ tal que $f(c)=g(c) \Leftrightarrow f(c)-g(c)=0$,

$f(3)-g(3) =$ __ >0 $\quad$ Pelo T. de Bolzano $\exists c\in[3,6]$ tal que $f(c)-g(c)=0$

$f(6)-g(6) =$ __ <0

$f'(x)-g'(x)=0 \Leftrightarrow f'(x)=g'(x) \Leftrightarrow x=2$.

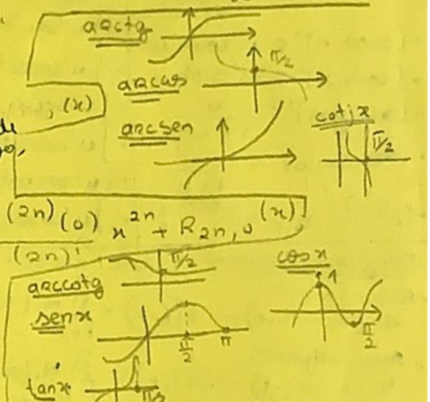
Como $x=2$ não pertence ao intervalo, a derivada ñ tem zeros, pelo T. de Rolle, não podem existir 2 zeros de $f(x)-g(x)$ em $[3,6]$, logo, só se intersetam 1 vez.

→ Aproximação linear: $\operatorname{sen} x \approx x$

- $\cos x = f(0) + f'(0)x + \frac{f''(0)}{2!} + \frac{f'''(0)}{3!}x^3 + \dots + \frac{f^{(2n)}(0)}{(2n)!}x^{2n} + R_{2n,0}(x)$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + (-1)^n \frac{x^{2n}}{(2n)!} + R_{2n,0}(x)$$

$\cos x \simeq 1$ (linear) $\quad$ e $\quad \cos x \simeq 1 - \frac{x^2}{2}$ (quadrática)



- Aproximar sen (0,1) com um erro inferior a 10^{-6}

$f(x) = \operatorname{sen} x$ em torno de $x=0$ (fórmula de Mac-Laurin: $\operatorname{sen} x = T_{n,0}(x) + R_{n,0}(x)$

$$= f(0) + f'(0) + \frac{f''(0)}{2!}x^2 + \dots \frac{f^{(n)}(0)}{n!}x^n + \frac{f^{(n+1)}(c)}{(n+1)!}x^{n+1}$$

$\operatorname{sen}(0,1) = T_{n,0}(0,1) + R_{n,0}(0,1)$

$$= f(0) + f'(0)0{,}1 + \frac{f''(0)}{2!}0{,}1^2 + \dots + \frac{f^{(n)}(0)}{n!}0{,}1^n + \frac{f^{(n+1)}(c)}{(n+1)!}0{,}1^{n+1}$$

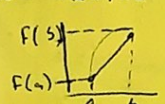
Atendendo que: $f(x) = \operatorname{sen} x$
derivar $f'''(x) \to f'''(0) = \cdot \Leftrightarrow \operatorname{sen} 0 = 0$ $\quad$ onde c é um nº estritamente entre 0 e 0,1
$\Leftrightarrow \operatorname{sen} x$

tem-se: $|\operatorname{sen}(0,1) - T_{n,0}(0,1)| = |R_{n,0}(0,1)| = \left|\frac{f^{(n+1)}(c)}{(n+1)!}0{,}1^{n+1}\right| \leq \frac{1}{(n+1)!\,10^{n+1}}$

$\operatorname{sen}(0,1) \simeq f(0) + f'(0)(0,1) + \frac{f''(0)}{2!}0{,}1^2 + \frac{f'''(0)}{3!}0{,}1^3 + \frac{f''''(0)}{4!}0{,}1^4 = 0{,}1 - \frac{1}{3!}0{,}3^3 = 0{,}0998$

Teorema de Lagrange

- Seja f uma função contínua em $[a,b]$, f é diferenciável em $]a,b[$, então, $\exists c\in\,]a,b[: f'(c) = \frac{f(b)-f(a)}{b-a}$



→ o declive da reta é igual ao declive da reta tangente em algum ponto c: as retas são paralelas

Integrais Racionais

$$f(x) = \frac{x+1}{x^3(x^2+1)}$$

$$= \frac{A}{x^3} + \frac{B}{x^2} + \frac{C}{x} + \frac{Dx+E}{x^2+1}$$

$$\to \int \frac{2}{x^2} = \int 2x^{-2} = 2\int \frac{1}{u'}\frac{x^{-2}}{u} = -\frac{2}{x}$$

Volumes

função de cima – função de baixo

- $V = \pi\int_a^b [f(x)]^2\,dx$ → eixo x

$V = \pi\int_a^b [f(x)-L]^2\,dx$ → eixo qualquer (ñ importa a ordem)

Intervalos de monotonia

$$f'(x) = \begin{cases} 2x+3 & \text{se } x\leq 0 \\ \frac{3}{3x+1} & \text{se } x>0 \end{cases}$$

– fazer as derivadas dos ramos

C.I

$f''(x) = 0 \Leftrightarrow (2x+3=0 \wedge x<0) \vee \left(\frac{3}{3x+1}=0 \wedge x>0\right)$

$\Leftrightarrow x = -\frac{3}{2}$

Pontos de inflexão

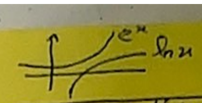
$f''(x) = 2 \quad x<0$

	$-\infty$	0	$+\infty$
$f''(x)$			
$f(x)$		ND	

Tem

Função par: $f(-x) = f(x)$
Função ímpar: $f(-x) = -f(x)$
• função composta: $(f \circ g)(x) = f(g(x))$
$D_{f\circ g} = \{x \in D_f : f(x) \in D_g\}$
$D_{f\circ g} = \{x \in \mathbb{R} : x \in D_f \wedge f(x) \in D_g\}$

• $(a-b)^2 = a^2 - 2ab + b^2$ • $|x| = \begin{cases} x & \text{se } x \geq 0 \\ -x & \text{se } x < 0 \end{cases}$
• $(a+b)^2 = a^2 + 2ab + b^2$
• $(a+b)(a-b) = a^2 - b^2$
• $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
$|x| < a \Leftrightarrow -a < x < a$
$|x| > a \Leftrightarrow x > a \vee x < -a$



Assíntotas Horizontais
$m = 0$ $y =$ ___
$\lim_{x \to \pm\infty} f(x)$

FACULDADE CIÊNCIAS
Departamento de Matemática

UNIVERSIDADE DA BEIRA INTERIOR
DEPARTAMENTO DE MATEMÁTICA
Licenciatura em Biotecnologia
Cálculo I
Ano Letivo 2022/2023

Nome: Nº:

$\int 2x e^{-x}$
$= 2\int \underbrace{x}_{u} \underbrace{e^{-x}}_{v'}$

Assíntotas não verticais
$\lim_{x\to+\infty} [f(x) - (mx+b)] = 0$
$m = \lim_{x \to \pm\infty} \frac{f(x)}{x}$
$b = \lim_{x \to \pm\infty} [f(x) - mx]$
substituir: $y = mx + b$
$\to D_f =]a,b[\to$ não tem
$\to$ calcula Df!
$\to D_f = \mathbb{R}\setminus\{a\}$ ou $\mathbb{R}$ ou $]-\infty, a[$ ou $]a, +\infty[$

• $\int x e^{cx^2} = \frac{1}{2c} e^{cx^2} + c$; • $\int e^u \times u' = e^u \to \int e^{3x-1}dx = \int 1 \times e^{3x-1}dx = \frac{1}{3}\int 3e^{3x-1}dx = \frac{1}{3}e^{3x-1}$; $\int e^x dx = x' e^x$

$\int 1 \times \frac{e^{3x-1}}{3}dx = \frac{1}{3}\int e^{3x-1}dx$

• $\int \frac{x'}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c$

• $\int u' u^n = \frac{u^{n+1}}{n+1}$ $\to \int \underbrace{(\cos(x))}_{u'} \times \frac{1}{3 \times \underbrace{(\operatorname{sen}(x))}_{u}^2}$

Ex: $\int \frac{\cos x}{(1+2\operatorname{sen} x)^3} = \frac{1}{2}\int 2\cos x \,(1+2\operatorname{sen} x)^{-3}$

• $\int \cos x = \operatorname{sen} x$
• $\int \operatorname{sen} x = -\cos x$
• $\int u' \cos u = \operatorname{sen} u$
• $\int_a^a f(x) = 0$
• $\int \frac{u'}{a^2+u^2} = \frac{1}{a}\operatorname{arctg}\frac{u}{a}$

$\frac{2+x}{x^3-3x^2} = \frac{2+x}{x^2(x-3)}$
$= \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-3}$

• $\int \frac{1}{x} = \ln|x|$
• $\int \frac{u'}{\sqrt{a^2-u^2}} = \operatorname{arcsen}\frac{u}{a}$; • $\int u' \sec^2 u = \operatorname{tg} u$
• $\int \frac{u'}{u} = \ln(u)$
• $\int x^\alpha = \frac{x^{\alpha+1}}{\alpha+1} \to \int \sqrt{x+1}\,dx = \int (x+1)^{\frac{1}{2}}dx = \frac{(x+1)^{\frac{1}{2}+1}}{\frac{1}{2}+1}$
• $\int \operatorname{arctg} x = x \operatorname{arctg}(x) - \frac{1}{2}\cancel{\log}\ln(x^2+1)$

LIMITES
• $\lim_{x\to 0} \frac{\ln(x+1)}{x} = 1$
• $\lim_{x\to 0} \frac{\operatorname{sen} x}{x} = 1$
• $\lim_{x\to 0} \frac{\tan x}{x} = 1$
• $\lim_{x\to+\infty} \left(1+\frac{1}{x}\right)^x = e$
• $\lim_{x\to 0} \frac{e^x - 1}{x} = 1$
• $\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$
• $\lim_{x\to+\infty} \frac{\ln x}{x} = 0$
• $\lim \frac{\operatorname{arcsen}}{} = 1$

1º Primitiva imediata?
2º Primitiva Racional:
a) - $f(x) = a + \frac{b}{x-c}$
$\begin{array}{c|c} \cdots & x-c \\ \hline b & a \end{array}$ ← grau inferior
b) $f(x) = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x-1)^2}$
$(x-1) = (A+B)x^2 + (B+A)x + C$
$1 \quad 1 \quad 0$
$\Leftrightarrow \begin{cases} A+B=0 \\ B+A=1 \\ C=1 \end{cases}$
3º Por partes:
$\int u v' = uv - \int u'v$
⊕ fácil

$f'(x_0) = \lim_{h\to 0^-} \frac{f(x_0+h) - f(x_0)}{h}$ ← Diferenciabilidade

$f'(x) = \frac{f(x) - f(x_0)}{x - x_0}$
$\lim_{x\to x_0^-}$ ___ e $\lim_{x\to x_0^+}$ ___
$f'(x_0^-) = f'(x_0^+)$, logo f é diferenciável em $\mathbb{R}$, pois nos intervalos (ver nos ramos) ___, resulta de operações entre funções diferenciáveis

COMPLETAR QUADRADOS
$x^2 + bx + c$
$= \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$

• $(\sqrt[n]{u})' = \frac{u'}{n\sqrt[n]{u^{n-1}}} \to (\sqrt{x})' = \frac{1}{2\sqrt{x}}$; • $(f(g))' = f'(g) \times g'$
• $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$
• $[\operatorname{sen}(u)]' = u' \cos u$; • $(\cos u)' = -u' \operatorname{sen} u$
• $(\tan u)' = u' \sec^2 u$
• $(f \times g)' = f'g + fg'$
$\frac{0}{0} \to \frac{(\ )'}{(\ )'}$
• $(f^n)' = n \times f^{n-1} \times f'$
• $(e^u)' = u' e^u \to e^0 = 1$; $e^{\ln\left(\frac{1}{t}\right)} = \frac{1}{t}$
• $(\operatorname{cotg} u)' = \frac{-u'}{\operatorname{sen}^2 u}$
• $(\ln u)' = \frac{u'}{u} \to$ • $\ln 1 = 0$ • $\ln e = 1$ • $\log\frac{1}{x} = \log x^{-1} = -\log(x)$
• $(\tan u)' = u' \sec^2 u$
• $\operatorname{cotan} u = -u' \operatorname{cosec}^2 u = -\frac{u'}{\operatorname{sen}^2 u}$
$d(\arccos u)' = \frac{-u'}{\sqrt{1-u^2}}$
• $(\operatorname{arcsen} u)' = \frac{u'}{\sqrt{1-u^2}}$
• $(\operatorname{arctg} u)' = \frac{u'}{1+u^2}$
• $(\operatorname{arccotan} u)' = -\frac{u'}{1+u^2}$

TRIGONOMETRIA
$\cos x = 0$
$\Leftrightarrow x = \frac{\pi}{2} + k\pi$
$\operatorname{sen} x = 0$
$\Leftrightarrow x = k\pi$
$\operatorname{arcsen}(1) = \frac{\pi}{2}$
$\operatorname{arcsen}(-1) = -\frac{\pi}{2}$
arcsen: $[-1,1] \to \mathbb{R}$, $y \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
arccos: $[-1,1] \to \mathbb{R}$, $y \to [0, \pi]$
• $\operatorname{sen}^2 x + \cos^2 x = 1$
• $\tan x = \frac{\operatorname{sen} x}{\cos x}$
• $\operatorname{sen}(2x) = 2\operatorname{sen}x\cos x$
• $\operatorname{sen}^2 x = \frac{1+\cos(2x)}{2}$
• $\sec x = \frac{1}{\cos x}$
• $\cos(2x) = \cos^2 x - \operatorname{sen}^2 x$
• $\cos^2 x = \frac{1+\cos(2x)}{2}$
• $\operatorname{sen}(\arccos x) = \sqrt{1-x^2}$
• $\operatorname{sen}(\operatorname{arctg} x) = \frac{x}{\sqrt{1-x^2}}$
• $\operatorname{sen}\left(\arccos\frac{1}{x}\right) = \frac{\sqrt{x^2-1}}{x}$
• $\operatorname{cotg}(\operatorname{arcsen} x) = \frac{\sqrt{1-x^2}}{x}$

θ	$\operatorname{sen}\theta$	$\cos\theta$	$\tan\theta$
$30°$ ($\frac{\pi}{6}$)	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$45°$ ($\frac{\pi}{4}$)	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$60°$ ($\frac{\pi}{3}$)	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

• $\operatorname{arctg}(0) = 0$
• $\operatorname{arctg}(1) = \frac{\pi}{4}$
• $\operatorname{arctg}(-\infty) = -\frac{\pi}{2}$

Teorema Bolzano
Uma vez que a função é contínua no intervalo $[a,b]$ e $f(a) \neq f(b)$ podemos concluir que, para qualquer valor K, $\exists c \in [a,b]: f(c) = K$
$f(a) \times f(b) < 0$ para $f: [a,b] \to \mathbb{R}$
logo $\exists c \in [a,b]: f(c) = 0$

Teorema de Taylor
$f(x) = \underbrace{f(a) + f'(a)(x-a)}_{\text{ap. linear}} + \frac{f''(a)}{2!}(x-a)^2 + \ldots \frac{f^n(a)(x-a)^n}{n!}$
aprox. quadrática
$R_n(x) = \frac{f^{n+1}(c)}{(n+1)!}(x-a)^{n+1}$
→ Se $a = 0$ → fórmula de Mac-Laurin

Intersetam-se apenas 1 vez em $[3,6]$?
Intersetam-se se $\exists c$ tal que $f(c) = g(c) \Leftrightarrow f(c) - g(c) = 0$
Pelo T. Bolzano $\exists c \in [3,6]$ tal que $f(c) - g(c) = 0$
$f(3) - g(3) =$ ___ > 0 ; $f'(x) - g'(x) = 0 \Leftrightarrow f'(x) = g'(x)$
$\Leftrightarrow x = 2$
$f(6) - g(6) =$ ___ < 0
Como $x = 2 \notin [3,6]$, a derivada não tem zeros pelo T. de Rolle, não pode existir 2 zeros de $f(x) - g(x)$ em $[3,6]$, logo só se intersetam 1x

Assíntotas verticais
→ calcular Df:
→ $D_f = \mathbb{R}$ → contínua, não tem
→ $D_f = \mathbb{R}\setminus\{a\}$ → a é ponto aderente
→ $\lim_{x\to a^\pm} f(x)$ → se der $\pm\infty$ em algum, não há $x = a$ A.V.